

# Negative refraction from quasi-planar chiral inclusions

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This letter proposes a quasi-planar chiral resonator suitable for the design of negative refractive index matamaterial. It is presented an analytical model for the determination of its polarizabilities, and the viability of negative refraction in chiral and racemic arrangements with the proposed inclusions is analyzed. The present analysis is expected to pave the way to the design of negative refractive index matamaterials made of a single kind of inclusions feasible from standard photo-etching techniques.

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The main aim of this letter is to explore the possibility of obtaining negative refraction from a random arrangement of quasi-planar chiral inclusions. Artificial bi-isotropic chiral media made of random arrangements of metallic chiral inclusions are known for long, after the former works of K. Lindmann [1]. More recently in [2] balanced (or *racemic*) mixtures of such type of inclusions were proposed as a way to obtain negative refractive index matamaterials. The general conditions for negative refraction of plane waves at the interface between ordinary and chiral media were analyzed in [3], and the focusing of circularly polarized light by a chiral slab was studied in [4]. The main advantage of chiral elements in order to provide negative refraction is that only one kind of inclusions is necessary to obtain negative values of  $\epsilon$  and  $\mu$ . An additional advantage would come from the application of conventional printed circuit fabrication techniques to manufacture such inclusions. For such purpose, a quasi-planar design would be desirable.

The proposed inclusion is shown in Fig. 1. It is the broadside-coupled version of the two turns spiral resonator (2-SR) previously proposed by some of the authors as a metamaterial element [5]. The analysis in that paper shows that the proposed element can be characterized by a quasi-static  $LC$  circuit, where  $L$  is the inductance of a single ring with the same radius and width as the inclusion, and  $C = 2\pi r C_{\text{pul}}$  is the total capacitance between the rings. However, there are two main differences between the structure of Fig. 1 and the 2-SR analyzed in [5]. First, due to the broadside coupling, the distributed capacitance between the rings can be made very large, which will reduce the electrical size of the inclusion near the resonance. Second, when the element is excited near the resonance, in addition to a strong magnetic dipole there also appears a strong electric dipole oriented parallel to the former one. This latter property comes from the strong electric field between the upper and lower rings that appears near the resonance.

Neglecting losses, and following the analysis in [5], the circuit equation for the total current in the element (i.e.,

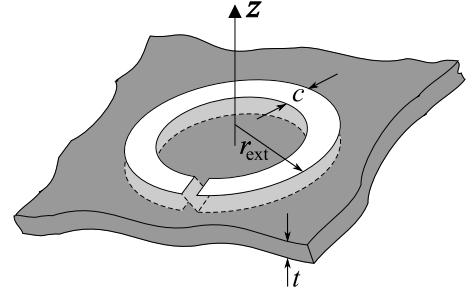


FIG. 1: The proposed inclusion is formed by two identical conducting rings, etched on both sides of a dielectric substrate, and connected by a via in order to obtain an helicoidal shape.

for the sum of the currents excited on both rings, which must be angle-independent [5]) is given by

$$\left(\frac{1}{j\omega C} + j\omega L\right) I = \Sigma, \quad (1)$$

where  $\Sigma$  stands for the external voltage excitation. For magnetic excitation:  $\Sigma = -j\omega\pi r^2 B_z^{\text{ext}}$ , where  $r$  is the mean radius of the inclusion. For electric excitation:  $\Sigma = t C_0 / C E_z^{\text{ext}}$ , where  $t$  is the substrate thickness and  $C_0$  is the total capacitance between the rings in the absence of the dielectric substrate [6]. From such equations, the following electric and magnetic moments excited in the inclusion when it is submitted to some external electric and/or magnetic fields can be obtained:

$$\begin{aligned} m_z &= \alpha_{zz}^{\text{mm}} B_z^{\text{ext}} - \alpha_{zz}^{\text{em}} E_z^{\text{ext}} \\ p_z &= \alpha_{zz}^{\text{ee}} E_z^{\text{ext}} + \alpha_{zz}^{\text{em}} B_z^{\text{ext}}, \end{aligned} \quad (2)$$

where

$$\alpha_{zz}^{\text{mm}} = \frac{\pi^2 r^4}{L} \left(\frac{\omega_0^2}{\omega^2} - 1\right)^{-1} \quad (3)$$

$$\alpha_{zz}^{\text{em}} = \pm j\pi r^2 t C_0 \frac{\omega_0^2}{\omega} \left(\frac{\omega_0^2}{\omega^2} - 1\right)^{-1} \quad (4)$$

$$\alpha_{zz}^{\text{ee}} = t^2 C_0^2 L \frac{\omega_0^4}{\omega^2} \left(\frac{\omega_0^2}{\omega^2} - 1\right)^{-1}, \quad (5)$$

with  $\omega_0 = \sqrt{1/LC}$  being the frequency of resonance. From (3)–(5) follows that

$$\alpha_{zz}^{mm} \alpha_{zz}^{ee} = -(\alpha_{zz}^{em})^2, \quad (6)$$

which will be useful in the following [7]. When  $N$  chiral inclusions are assembled in a random way, the resulting medium becomes bi-isotropic with constitutive relations given by

$$\mathbf{D} = \varepsilon_0 \varepsilon_r \mathbf{E} + j\sqrt{\varepsilon_0 \mu_0} \kappa \mathbf{H}; \quad \varepsilon_r = (1 + \chi_e) \quad (7)$$

$$\mathbf{B} = -j\sqrt{\varepsilon_0 \mu_0} \kappa \mathbf{E} + \mu_0 \mu_r \mathbf{H}; \quad \mu_r = (1 + \chi_m). \quad (8)$$

The electric,  $\chi_e$ , magnetic,  $\chi_m$ , and cross,  $\kappa$ , susceptibilities are related to the inclusion polarizabilities through

$$\chi_e = \frac{N}{\Delta \varepsilon_0} \frac{\alpha_{zz}^{ee}}{3}; \quad \chi_m = \frac{N \mu_0}{\Delta} \frac{\alpha_{zz}^{mm}}{3}; \quad \kappa = \pm j \frac{N}{\Delta} \sqrt{\frac{\mu_0}{\varepsilon_0}} \frac{\alpha_{zz}^{em}}{3}, \quad (9)$$

where the factor  $1/3$  arises from the random arrangement, and  $\Delta$  is a common factor that depends on the homogenization procedure. From (6) and (9) follows that

$$\chi_e(\omega) \chi_m(\omega) = [\kappa(\omega)]^2. \quad (10)$$

As is well known, the general dispersion equation for plane waves in lossless chiral media is

$$k = \pm k_0 \left( \sqrt{(1 + \chi_e)(1 + \chi_m)} \pm \kappa \right), \quad (11)$$

where  $k_0 = \omega \sqrt{\varepsilon_0 \mu_0}$ . The four solutions of (11) correspond to right- and left-hand circularly polarized waves, depending on the sign of  $\kappa$ . In order to avoid complex solutions of (11), and therefore forbidden frequency bands for plane wave propagation, it would be desirable that  $\chi_e(\omega) = \chi_m(\omega)$ . According to (10) this implies that

$$\chi_e = \chi_m = |\kappa|. \quad (12)$$

The general condition for backward-wave propagation is found to be [3]

$$\sqrt{\varepsilon_r \mu_r} \pm \kappa < 0, \quad (13)$$

where the sign of the square root must be chosen negative if both  $\varepsilon_r$  and  $\mu_r$  are negative. According to (13), if  $\kappa^2 > |\varepsilon_r \mu_r|$  only one of solutions of (11) can be a backward-wave and, therefore, will experience negative refraction at the interface with an ordinary media. This is indeed the case when (12) is satisfied and  $\chi_e, \chi_m$ , are both negative. In such case, negative refraction will take place for only one of the eigenmodes of (11), provided that  $\chi_e = \chi_m = |\kappa| < -0.5$ . This condition is less restrictive than the condition for ordinary media (for instance, for a balanced mixture of inclusions of opposite helicity), namely,  $\chi_e, \chi_m < -1$ . The price to pay for this

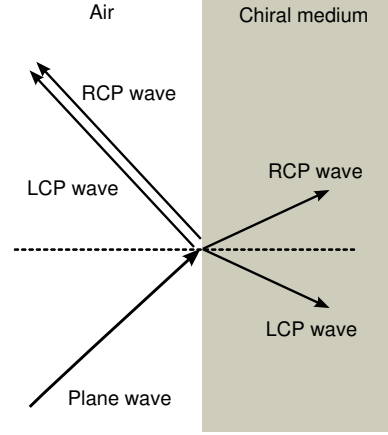


FIG. 2: Illustration of the negative refraction of a linearly polarized wave at the interface with a chiral metamaterial made of inclusions as that shown in Fig. 1. Only one of the two eigenwaves that can propagate in the chiral medium shows negative refraction, and the reflected wave is elliptically polarized.

enlargement of the bandwidth is that only one of the solutions of (11) shows negative refraction. Such scenario is illustrated in Fig. 2, where an incident linearly polarized wave is considered.

Returning now to the inclusions, it is found from (9) that condition (12) is satisfied provided that

$$c^2 \alpha_{zz}^{ee}(\omega) = \alpha_{zz}^{mm}(\omega) = \pm j c \alpha_{zz}^{em}(\omega), \quad (14)$$

where  $c$  is the velocity of light in vacuum. In principle, this condition is compatible with (3)–(5). Actually, we have tried to obtain a particular design satisfying such condition by using the analytical expressions for  $L$  and  $C_{\text{pul}}$  reported in [8]. A substrate with permittivity similar to vacuum (a foam for instance) was chosen in order to simplify computations. With this substrate ( $\epsilon = \epsilon_0$ ) a suitable design is: width of the strips  $c = 2$  mm, external radius  $r_{\text{ext}} = r + c/2 = 5$  mm, and separation between strips  $t = 2.35$  mm. Following [8], the frequency of resonance of the proposed configuration should be about 2.3 GHz. It gives an electrical size of  $\sim \lambda/13$  for the inclusion, which is acceptable for a practical metamaterial design. In order to check our analytical results, the electric and magnetic polarizabilities of the inclusions have been numerically determined following the procedure described in [9]. This procedure mainly consists in placing the particle inside a TEM waveguide and to compute the polarizabilities from the reflection and transmission coefficients of the loaded waveguide (see [9] for more details). The results for the meaningful quantities  $\mu_0 \alpha_{zz}^{mm}$  and  $\alpha_{zz}^{ee}/\epsilon_0$  are shown in Fig. 3. These results clearly confirms the conclusions of our analytical model. The cross polarizations cannot be numerically determined following the method described in [9]. However, the equality between the meaningful quantities  $\mu_0 \alpha_{zz}^{mm}$

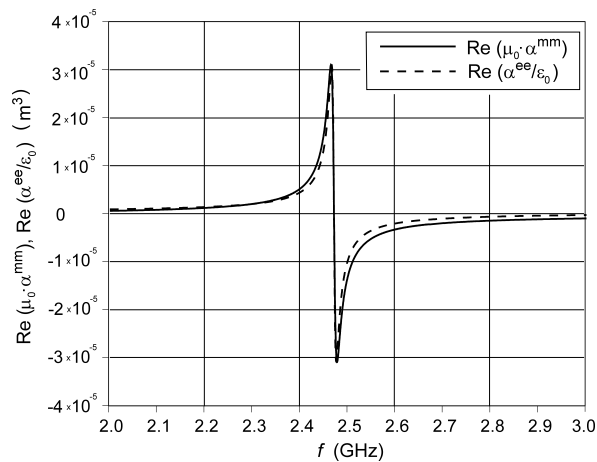


FIG. 3: Numerical determination of  $\mu_0\alpha_{zz}^{mm}$  and  $\alpha_{zz}^{ee}/\epsilon_0$  for the inclusion shown in Fig.1 with the parameters given in the text.

and  $|\sqrt{\mu_0/\epsilon_0}\alpha_{zz}^{em}|$  can be shown by comparing the reflection coefficient for the co- and the cross-polarized waves when the particle is placed inside a metallic waveguide of square cross-section. If the waveguide is wide enough, so as the wave impedance approaches that of free space, and the particle is placed with its axis perpendicular to the waveguide walls, the equality of both reflection coefficients implies the equality of the above quantities. Numerical calculations (not shown) made with the commercial electromagnetic solver CST Microwave Studio confirms this prediction.

In order to evaluate the frequency bandwidth for negative refraction in a metamaterial made of a random arrangements of the proposed inclusions, the electric susceptibility  $\chi_e$  of such medium has been computed from (9) with  $\Delta = 1$ . Although this approximation is rather rough, it is clear from the general form of (9) that any other homogenization procedure (for instance, a generalized Clausius-Mossotti one) would give similar qualitative results. The dimensions and characteristics of the inclusions are those previously reported, and the number of inclusions per unit volume is  $N = (12)^{-3} \text{ mm}^{-3}$ . Both the analytical and the numerical results obtained from the data of Fig. 3 are shown in Fig. 4. From the analysis and the numerical results reported in the previous paragraphs directly follows that the curves (not shown) for the magnetic  $\chi_m$  and the cross susceptibility  $\kappa$  must be quite similar. Although some differences appear between the analytical and numerical results shown in Fig. 4, its qualitative agreement is apparent. In both cases a significant negative refraction frequency band appears for both the random and the racemic mixtures. As it was already mentioned, such frequency bands are limited by the straight lines  $\chi_e = -0.5$  and  $\chi_e = -1$  respectively (see Fig.4).

In summary, the feasibility of manufacturing negative refractive index metamaterials from a random arrange-

ment of chiral quasi-planar inclusions has been analyzed. It has been proposed an specific design with the advantage of being easily manufactured from standard photo-etching techniques. Also it has been shown that such design provides the necessary behavior for all the resonant polarizabilities in order to produce a significant negative refractive index bandwidth near the resonance.

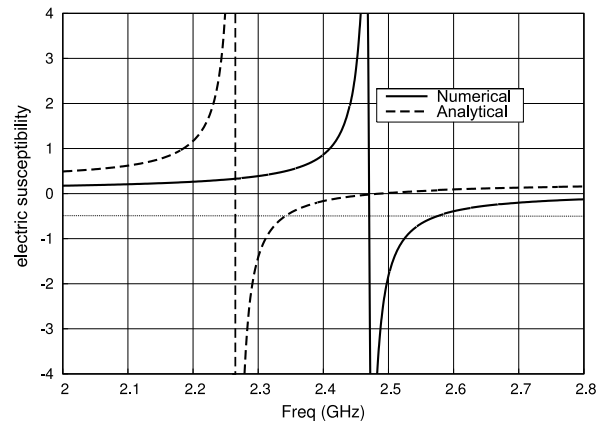


FIG. 4: Analytical and numerical results for the electric susceptibility  $\chi_e$  of a random arrangement of chiral inclusions as those shown in Fig. 1. The parameters of the inclusions are given in the text and are the same as in Fig. 3. The average volume per inclusion is  $V = 12^3 \text{ mm}^3$ .

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- [6] The factor  $C_0/C$  appears because, when a parallel-plate capacitor is excited by a normal external field, the electric field inside the capacitor is just the external one multiplied by the above factor.
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